Math 315 Sections 1 and 2 Test 1 Name _____
8–10 February 2007 Show relevant work!
D. Wright

1. State the Axiom of Completeness and use it to prove that a monotone increasing sequence converges to its least upper bound.

2. Show that the rational numbers are countable.

3. State the Schroeder-Bernstein Theorem.

4. Show that if $\lim a_n = a$ and $\lim b_n = b$, then $\lim a_n b_n = ab$.

5. Show that the Nested Interval Property implies the Axiom of Completeness.

6. Prove that the sequence defined by $x_1 = 3$ and $x_{n+1} = \dfrac{1}{4 - x_n}$ converges and find its limit.

7. Show that if $0 < r < 1$, then $\lim r^n = 0$.

8. Define what it means for a series $\sum a_n$ to converge.

9. Show that if $\sum a_n$ converges, then $\lim a_n = 0$.

10. Define what it means for a sequence to be Cauchy and show that a Cauchy sequence converges.